Answers to the Problems in Chapter 2

Problem 2.1.

You may find it helpful to read '*motion in a circle*' in Box 2.1.

Let the masses of the planet and of the sun be m and M respectively and suppose that the planet circles the sun with a velocity v in an orbit of radius R. Then, according to Newton, the force of attraction, F, between the planet and the sun is GmM/R^2, and this force induces an acceleration, a, of the planet toward the sun of v^2/R.

Again, according to Newton, $a = F/m$, so that we have the equation:

$$\frac{v^2}{R} = \frac{GmM}{mR^2} \quad \text{or} \quad v^2 = \frac{GM}{R}$$

The time required for one orbit, τ, is:

$$\tau = \frac{2\pi R}{v} = 2\pi R\sqrt{\frac{R}{GM}} \quad \text{or} \quad \tau^2 = \frac{4\pi^2 R^3}{GM}$$

Problem 2.2.

If the radius of the satellite's orbit, measured from the centre of the earth, is R m, then the velocity, v, of the satellite in its orbit, which must have a period of 24 hours, is $2\pi R/(24\times3600)$ ms^{-1}. We have shown in problem 2.1 that:

$$v^2 R = GM \quad \text{or} \quad R^3 = \frac{GM\times(24\times3600)^2}{4\pi^2} = \frac{5.983\times6.673\times7.465\times10^{22}}{39.478} = 75.494\times10^{21}\ \text{m}^3$$

Therefore, $R = 4.226\times10^7$ m and the altitude above the earth's surface is 35882 km.

The satellite's velocity is:

$$v = \sqrt{\frac{GM}{R}} = \sqrt{\frac{5.983\times6.673\times10^{13}}{4.266\times10^7}} = 3.159\times10^3\ \text{ms}^{-1}$$

Problem 2.3.

The problem refers to Figure 2.6, of course, not 2.5. My apologies!

Clearly, only a rather crude estimate of W(Na) and W(Li) is possible with this small figure, but we use Equation 2.6.2 which says that ν_0 is the frequency for which the energy, $E_0 = h\nu_0$, is just sufficient to eject an electron from the metal surface, i.e. is equal to W, the work function. At ν_0 the electron emerges with zero kinetic energy, corresponding to a retarding potential of zero. Therefore, we draw a horizontal line across the graph at retarding potential = 0 and estimate the value on the frequency axis where this line intersects the plots for Li and Na. We find that $\nu_0(\text{Na}) \approx 1050\times10^{12}$ Hz and $\nu_0(\text{Li}) \approx 930\times10^{12}$ Hz, so that, using Equation 2.6.2;

$W(\text{Na}) = 6.626\times10^{-34}\times1050\times10^{12} = 6957.3\times10^{-22}$ J or 4.19×10^{5} J mol^{-1}.
$W(\text{Li}) = 6.626\times10^{-34}\times930\times10^{12} = 6162.2\times10^{-22}$ J or 3.71×10^{5} J mol^{-1}.

Problem 2.4.

We need the equation $E = \frac{1}{2}mv^2 = h\nu = hc/\lambda$, where c is the velocity of light.
$E = \frac{1}{2}mv^2 = 0.5\times58.0\times10^{-3}\times(200.0\times10^{3}/3600)^2 = 89.506$ J.
Therefore, $\lambda = hc/E = 6.626\times10^{-34}\times2.998\times10^{8}/89.506 = 2.219\times10^{-27}$ m

Problem 2.5.

By definition, an electron accelerated by a potential of 1 V acquires an energy of 1 eV (electron volt). Therefore, the energies of the accelerated electrons ranged from 10 to 60 keV. In Table 8.5 we find that 1 eV = 0.160×10^{-18} J so that from the equation $\lambda = hc/E$ we have:
For 10 eV, $\lambda = 6.626\times10^{-34}\times2.998\times10^{8}/(10\times0.160\times10^{-18}) = 12.4\times10^{-9}$ m.
For 60 eV, $\lambda = 6.626\times10^{-34}\times2.998\times10^{8}/(60\times0.160\times10^{-18}) = 2.07\times10^{-9}$ m.

Problem 2.6.

The expansion of the exponential function is given in Box 3.1.

$\exp(ax) \equiv e^{-ax} = 1 + ax + (ax)^2/2! + (ax)^3/3! + \ldots$

If we use this expansion in Equation2.5.4b we have:

$$\rho(\nu) = \frac{8\pi h\nu^3}{c^3} \times \frac{1}{\{1 + (h\nu/kT) + (h\nu/kT)^2/2! + (h\nu/kT)^3/3! + \ldots - 1\}}$$

and when $h\nu \ll$ kT we may neglect all the terms in the expansion after the first two so that our expression for $\rho(\nu)$ becomes:

$$\rho(\nu) = \frac{8\pi h\nu^3}{c^3} \times \frac{1}{\{1 + (h\nu/kT) - 1\}} = \frac{8\pi h\nu^3}{c^3} \times \frac{kT}{h\nu} = \frac{8\pi kT}{c^3}\nu^2$$